hep-th/0403236

Trans-Planckian corrections to the primordial spectrum in the infra-red *and* the ultra-violet

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(Dated: February 1, 2008)

Due to the tremendous red-shift that occurs during the inflationary epoch in the early universe, it has been realized that trans-Planckian physics may manifest itself at energies much lower than the Planck energy. The presence of a fundamental scale suggests that local Lorentz invariance may be violated at sufficiently high energies. Motivated by this possibility, recently, different models that violate Lorentz invariance locally have been used to evaluate the trans-Planckian corrections to the inflationary density perturbation spectrum. However, certain astrophysical observations seem to indicate that local Lorentz invariance may be preserved to extremely high energies. In such a situation, to study the trans-Planckian effects, it becomes imperative to consider models that *preserve* local Lorentz invariance *even as they contain a fundamental scale*. In this work, we construct one such model and evaluate the resulting spectrum of density perturbations in the power-law inflationary scenario. While our model reproduces the standard spectrum on small scales, it *naturally* predicts a suppression of power on large scales. In fact, the spectrum we obtain has some features which are similar to the one that has recently been obtained from non-commutative inflation. However, we find that the amount of suppression predicted by our model is *far less* than that is required to fit the observations. We comment on the fact that, with a suitable choice of initial conditions, our approach can lead to corrections at the infra-red *as well as* at the ultra-violet ends of the spectrum.

PACS numbers: 98.80.Cq, 04.62.+v

I. INTRODUCTION AND MOTIVATION

Inflation, a period of accelerated expansion in the high-energy phase of the universe, is currently considered to be the best paradigm for describing the early stages of the universe [1, 2]. The success of the inflationary paradigm rests on its ability to explain not only the homogeneity of the background, but also the characteristics of the inhomogeneities superimposed upon it. The inflationary epoch magnifies the tiny fluctuations in the quantum fields present at the beginning of the epoch into classical perturbations that leave an imprint as anisotropies in the cosmic microwave background (CMB). These anisotropies in turn act as seeds for the formation of the large-scale structure that we observe at the present time as galaxies and clusters of galaxies. With anisotropies in the CMB being measured with higher and higher precision, we are currently able to test the predictions of inflation better and better.

During the last couple of years or so, considerable amount of attention has been devoted to examining the possibility that Planck scale physics may leave a measurable imprint on the CMB [3]–[14]. There are three reasons that have led to such an enormous interest in the literature. Firstly, there is no unique model of inflation [2]. In many versions of inflation, most notably in chaotic inflation, the period of inflation lasts sufficiently long so that the comoving length scales that are of cosmological interest today would have emerged from sub-Planckian length scales at the beginning of inflation. Hence, in principle, quantum gravitational effects should have left their signatures on the primordial spectrum of perturbations. Secondly, in the simplest models of inflation, the scale of the vacuum energy during the period of exponential expansion is assumed to be $\sim 10^{16}\,\mathrm{GeV}$, while the rate of the exponential expansion, viz. H, is considered to be $\sim 10^{14}\,\mathrm{GeV}$. These enormous energies suggest that during the inflationary epoch, various (as yet, unknown?) high-energy processes could have been activated which would have left their signatures on the primordial perturbation spectrum. These signatures on the perturbation spectrum in turn will leave their imprints on the CMB. Recent measurements of the CMB anisotropies by, say, WMAP [15], strongly indicate a primordial spectrum that is nearly scale-invariant, just as the inflationary scenario predicts. Therefore, further precise and accurate measurements of the CMB anisotropies by experiments such as PLANCK [16] can, in principle, provide us with the form of the corrections to the scale-invariant primordial perturbation spectrum. Lastly and, more importantly, the first results of the temperature-temperature

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correlation spectrum of the WMAP data show that the power in the quadrupole and (to a lesser extent) the octopole moment of the CMB are *lower than* as expected in the best fitting cold dark matter models [15]. The deficit of power in the quadrapole moment of the CMB power spectrum can not be explained within the context of the standard inflationary models (unless these models are fine-tuned [17]) and suggests a possible signature of the Planck scale physics [18].

Several groups have pointed out to these possibilities and, broadly, there have been two approaches in the literature in order to study these effects. In the first approach, which is now commonly referred to as the minimal trans-Planckian approach, the specific nature of trans-Planckian physics is not assumed, but is rather described by the boundary conditions imposed on the mode at the cut-off scale [10, 11]. In the second approach, one incorporates quantum gravitational effects by introducing the fundamental length scale into the standard field theory in a particular fashion and, it should be emphasized here that, the resulting modified theory may not even preserve local Lorentz invariance. Indeed, most of the attempts in this direction have involved models which *break* local Lorentz invariance. Such models include those which introduce non-linear dispersion relations [3–7] as well as the approaches which utilize the generalized uncertainty principle [8] and non-commutative geometry [9, 18]. (For a recent review on these various approaches, see Ref. [14].)

However, theoretically, there exists no apriori reason to believe that Lorentz invariance may be broken at the scales of inflation. More importantly, recent observations of photons inferred to arise from synchrotron emission off electrons in the Crab nebula have placed bounds on dispersive corrections involving additional powers of the particle momentum to be at or above the Planck scale [19]. In such a situation, in order to study the trans-Planckian effects on the primordial perturbation spectrum, it becomes important that we also consider models which preserve Lorentz invariance *even as they contain a fundamental scale*. In this work, we consider one such model and evaluate the resulting spectrum of density perturbations during inflation in this model.

The model we shall consider is as follows: We shall *assume* that, due to the Planck-scale effects, the standard $k-$space propagator for a massless scalar field in the Minkowski vacuum is modified in a particular manner. We shall introduce the high-energy scale $k_{\rm c}$ into the $k-$space propagator in a Lorentz invariant manner and, as we shall point out later, the modification we consider can be said to be minimal in nature. We find that the resulting modified Wightman function can be expressed as the difference of the Wightman functions of the massless field and a massive field of mass $k_{\rm c}$. We shall further assume that the form of the modified Wightman function in the $x-$space remains the same in an inflationary background as well, i.e. it is the difference of the massless and the massive Wightman functions. We then use this modified Wightman function to evaluate the corrections to the spectrum of density perturbations for the power-law inflationary scenario. We show that our model naturally predicts a suppression of power at the large scales. We find that the modified spectrum we obtain has some resemblance to the spectrum that has been obtained in the non-commutative inflationary scenario [18]. However, the amount of suppression predicted by our model turns out to be *far less* than that seems to be required to fit the WMAP data.

The rest of the paper is organized as follows. In Section (II), we outline the model we shall consider, present the motivations behind the model and also point out its attractive features. In Section (III), we shall briefly review the spectrum of fluctuations in the standard power-law inflationary scenario. In Section (IV), we shall evaluate the corrections to the spectrum of fluctuations using our model for the power-law inflation. Finally, in Section (V), we shall summarize the results of our analysis and also discuss its implications.

Before we proceed, a few comments on the notations we shall use are in order. The metric signature we shall adopt is $(+,-,-,-)$, we shall set $\hbar = c = 1$ and we shall denote the set of four coordinates x^μ simply as $\tilde{x}$. Also, the quantum field Ψ we shall consider will be a minimally coupled scalar field.

II. THE MODEL

In the inflationary scenario, the primordial perturbation spectrum is given by the Fourier transform of the Wightman function of a quantized, massless scalar field in the inflating background [1, 2]. Therefore, in order to understand the effects of Planck-scale physics on the perturbation spectrum, we need to understand as to how quantum gravitational effects will modify the propagator of a scalar field in an inflationary background. However, due to the lack of a clear understanding of the Planck-scale effects, often, one is forced to consider models constructed by hand—models which are supposed to be *effective theories* obtained by integrating out the gravitational degrees of freedom.

For reasons we had outlined in the introductory section, in this work, we would like to consider a model that is locally Lorentz invariant and also contains the high-energy scale. Recall that, in the Minkowski vacuum in flat space-time, the propagator for a massless scalar field in $k-$space is given by

$$G_0^+(k) = \left(\frac{1}{\bar{k}^2}\right), \tag{1}$$

where $\bar{k}^2 \equiv (k^\mu k_\mu)$. Planck-scale effects are expected to modify the propagator even in flat space-time (see, for e.g., Refs. [20] and references therein). If we require that the effective theory describing the modified propagator be Lorentz invariant, then, in flat space-time, the modified propagator can only be a function of $\bar{k}$. We shall

assume that the standard propagator (1) for the massless field is modified to

$$G_{\rm M}^{+}(k) = \left(\frac{1}{\bar{k}^2\left[1-\alpha^2(\bar{k}/k_{\rm c})^2\right]}\right), \tag{2}$$

where α^2 is an arbitrary positive constant fixed by the complete theory and $k_{\rm c}$ denotes the cut-off scale which we shall assume to be, say, three to five orders of magnitude above the Hubble scale during inflation. Such a modification can be considered to be minimal as it contains only terms of order $\bar{k}^4$ and does not contain any higher order terms. (We introduce the $\bar{k}^4$ term rather than the immediately higher order $\bar{k}^3$ term, as it makes the calculation of the resulting perturbation spectrum far more tractable.) Note that the modified propagator $G_{\rm M}^{+}(k)$ can be expressed as

$$G_{\rm M}^{+}(k) = \left(\frac{1}{\bar{k}^2}\right) - \left(\frac{1}{\bar{k}^2 - k_{\rm c}^2}\right), \tag{3}$$

where we have absorbed α into $k_{\rm c}$. Then, in $x-$space, the Wightman function corresponding to the above $k-$space propagator is given by

$$G_{\rm M}^{+}\left(\tilde{x}, \tilde{x}'\right) = G_{0}^{+}\left(\tilde{x}, \tilde{x}'\right) - G_{k_{\rm c}}^{+}\left(\tilde{x}, \tilde{x}'\right), \tag{4}$$

where $G_{0}^{+}\left(\tilde{x}, \tilde{x}'\right)$ and $G_{k_{\rm c}}^{+}\left(\tilde{x}, \tilde{x}'\right)$ denote the the Wightman functions of the massless field and a massive field of mass $k_{\rm c}$. In order to evaluate the resulting modification to the primordial perturbation spectrum, we shall assume that the above modified Wightman function retains the same form (i.e. it is the difference of the Wightman functions of massless and massive fields) in the inflationary background as well.

Apart from the fact that we required a minimal (and tractable) Lorentz invariant model, the other motivation for the above model is as follows: The divergence structure of quantum field theory is expected to vastly improve when the quantum gravitational effects have been taken into account. In particular, propagators are expected to be finite in the limit when the two space-time points coincide [20]. Though, the modified Wightman function $G_{\rm M}^{+}(\tilde{x}, \tilde{x}')$ we have constructed will not actually be finite as $\tilde{x} \to \tilde{x}'$, it certainly turns out to be far less divergent than the original Wightman function. (In this sense, our approach can be said to be motivated by the Pauli-Villars regularization procedure.) For instance, in the Minkowski vacuum, while the original Wightman function will diverge as 0^{-2} in the coincident limit, the modified function (4) will diverge as $\ln 0$ (see, for instance, Ref. [21]). Moreover, we should add that a similar approach has been considered earlier to study the effects of trans-Planckian physics on Hawking radiation from black holes [22].

At this stage of our discussion, it is important that we compare our model with the models that introduce non-linear dispersion relations to study the trans-Planckian effects on the primordial perturbation spectrum [3–7]. In these models, there has been the growing issue of the choice of the vacuum state and the back-reaction of the trans-Planckian modes on the inflationary background. It has been shown that the back-reaction effects during inflation become important for those dispersion relations which induce large corrections to the scale-invariant spectrum [6]. This feature essentially arises due to the fact that adiabatic evolution can occur only if the dispersion relations either grow or, at least, reach a plateau for large values of the wavenumber [4, 5]. In contrast, in our model, the modified propagator and the resulting modification to the perturbation spectrum involve modes of massless and massive fields which always evolve adiabatically.

Before proceeding to the technical aspects, it is necessary that we clarify certain conceptual issues related to our model. In the conventional models of quantum field theory (and also in the other models that incorporate quantum gravitational corrections such as the dispersive field theory models), the Lagrangian is a function of the fields and their first derivatives in time. However, our model contains second order time derivatives in the Lagrangian. It can be easily shown that, in flat space-time, the Feynman propagator corresponding to the modified Wightman function (4) satisfies the equation (see, for e.g., Ref. [21]; in this context, also see, Refs. [22, 23])

$$\left[\Box + \left(\frac{1}{k_{\rm c}^2}\right)\Box^2\right] G_{\rm M}^{F}(\tilde{x}, \tilde{x}') = -\,\delta^{(4)}\left(\tilde{x} - \tilde{x}'\right) \tag{5}$$

which in turn can be obtained from the following action

$$S[\Psi] = \int d^4x\, \left[\left(\frac{1}{2}\right)\partial_\mu\Psi\,\partial^\mu\Psi - \left(\frac{1}{2\,k_{\rm c}^2}\right)(\Box\Psi)^2\right], \tag{6}$$

where Ψ denotes a scalar field and $k_{\rm c}$ the cut-off scale.

Theories described by higher derivative Lagrangians are well-known to have unsatisfactory properties [23, 24]. They are known to contain additional degrees of freedom, the energy is not bounded from below and, also, the solutions to the equations of motion are not uniquely determined by the initial values of the fields and their first time derivatives. Clearly, these features can be considered to be unwelcome when we are dealing with Lagrangians that are expected to parameterize *small* deviations from a well-understood theory.

In such a situation, we shall adopt the following point-of-view concerning our model. As mentioned earlier, we consider our model to be an effective theory of the inflaton field obtained by integrating out the Planck scale effects. Non-locality often arises in such effective theories. Effective theories in which the non-locality can be regulated by a small parameter are known to have perturbation expansions with higher derivatives [24]. We believe that our model is the leading term in the perturbation expansion in the small parameter $(1/k_{\rm c})$ of a non-local effective theory. Moreover, we expect that, in the complete theory, we may not encounter the difficulties that we mentioned in the previous paragraph.

III. SPECTRUM OF PERTURBATIONS FROM INFLATION—THE STANDARD RESULT

In this section, we shall briefly rederive the standard spectrum of perturbations in the power-law inflationary scenario.

Consider a flat Friedmann universe described by the line-elements

$$ds^2 = \left(dt^2 - a^2(t)\, d\mathbf{x}^2\right) = a^2(\eta)\left(d\eta^2 - d\mathbf{x}^2\right), \qquad (7)$$

where t is the cosmic time, $a(t)$ is the scale factor and $\eta = \int [dt/a(t)]$ denotes the conformal time. Power-law inflation corresponds to the situation where the scale factor $a(t)$ is given by

$$a(t) = (a_0\, t^p), \qquad (8)$$

where $p > 1$. In terms of the conformal time η, this scale factor can be written as

$$a(\eta) = (-\mathcal{H}\,\eta)^{(\beta+1)}, \qquad (9)$$

where β and $\mathcal{H}$ are given by

$$\beta = -\left(\frac{2p-1}{p-1}\right) \quad \text{and} \quad \mathcal{H} = \left[(p-1)\, a_0^{1/p}\right]. \qquad (10)$$

Note that $\beta \le -2$ and $\mathcal{H}$ denotes the characteristic energy scale associated with inflation. Also, the case of exponential expansion corresponds to $\beta = -2$ with $\mathcal{H}$ actually being equal to the Hubble scale.

Metric fluctuations (both scalar as well as the tensor perturbations) during the inflationary epoch can be modeled by a massless, minimally coupled scalar field, say, Ψ [1, 25]. The field Ψ propagating in a background described by the line-elements (7) satisfies the following Klein-Gordon equation:

$$\Box\Psi \equiv \left(\frac{\partial^2\Psi}{\partial\eta^2}\right) + \left(\frac{2}{a}\right)\left(\frac{da}{d\eta}\right)\left(\frac{\partial\Psi}{\partial\eta}\right) - \nabla^2\Psi = 0. \quad (11)$$

The symmetry of the Friedmann metric allows us to decompose the normal modes of the scalar field as

$$\psi_{\mathbf{k}}(\tilde{x}) = \left(\frac{1}{(2\pi)^{3/2}}\right)\left(\frac{\mu_k(\eta)}{a(\eta)}\right) e^{i\mathbf{k}\cdot\mathbf{x}}, \qquad (12)$$

where $\mathbf{k}$ is the comoving wave vector and the function μ_k satisfies the differential equation

$$\mu_k'' + \left[k^2 - \left(\frac{a''}{a}\right)\right]\mu_k = 0 \qquad (13)$$

with the primes denoting differentiation with respect to η and $k = |\mathbf{k}|$.

On quantization, the scalar field can be expressed in terms of the normal modes $\psi_{\mathbf{k}}(\tilde{x})$ as follows:

$$\hat{\Psi}\,(\eta, \mathbf{x}) = \int d^3\mathbf{k}\, \left[\hat{a}_{\mathbf{k}}\, \psi_{\mathbf{k}}(\tilde{x}) + \hat{a}_{\mathbf{k}}^{\dagger}\, \psi_{\mathbf{k}}^{*}(\tilde{x})\right], \qquad (14)$$

where the creation and the annihilation operators $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^{\dagger}$ obey the usual commutation relations. As the perturbations are assumed to be induced by the fluctuations in the free quantum field $\hat{\Psi}$, the power spectrum as well as the statistical properties of the perturbations are entirely characterized by the Wightman functions of the quantum field. Therefore, the power spectrum of the perturbations per logarithmic interval, viz. $\left[k^3\, \mathcal{P}_\Psi(k)\right]$, is given by [1, 2]

$$\int_0^\infty \left(\frac{dk}{k}\right)\, \left[k^3\, \mathcal{P}_\Psi(k)\right] = \langle 0|\hat{\Psi}^2(\eta, \mathbf{x})|0\rangle = G_0^{+}\,(\tilde{x}, \tilde{x}), \qquad (15)$$

where $|0\rangle$ is the vacuum state (defined as $\hat{a}_{\mathbf{k}}|0\rangle = 0\ \forall\ \mathbf{k}$), $G_0^{+}(\tilde{x}, \tilde{x}')$ denotes the Wightman function of the massless quantum field $\hat{\Psi}$ and the spectrum is to be evaluated when the modes leave the Hubble radius. Using the decomposition (14), the perturbation spectrum per logarithmic interval can then be written in terms of the modes μ_k as

$$\left[k^3\, \mathcal{P}_\Psi(k)\right] = \left(\frac{k^3}{2\pi^2}\right)\left(\frac{|\mu_k|}{a}\right)^2 \qquad (16)$$

and the expression on the right hand side is to be evaluated when the physical wavelength $(k/a)^{-1}$ of the mode corresponding to the wavenumber $\mathbf{k}$ equals the Hubble radius H^{-1}, where $H = (a'/a^2)$. In the remainder of this section, we shall evaluate the spectrum of perturbations in power-law inflation[1].

For the case of power-law inflation, the exact solution to the differential equation (13) is well-known. Nevertheless, due to its utility in the next section (where we need to evaluate the modes of a massive field in the WKB approximation), we shall rewrite the differential equation (13) in terms of a new set of independent and dependent variables (x, u_k) which are related to the old set (η, μ_k) by the relations [26, 27]

$$x = \ln\left(\frac{\beta+1}{k\eta}\right) \qquad (17)$$

and

$$\mu_k = \mathrm{e}^{-(x/2)}\, u_k. \qquad (18)$$

In terms of the new variables, the differential equation (13) can be written as

$$\left(\frac{d^2u_k}{dx^2}\right) + \left[(\beta+1)^2\, \mathrm{e}^{-(2x)} - \left(\beta + \frac{1}{2}\right)^2\right] u_k = 0 \quad (19)$$

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[1] In this work, we do not distinguish between the scalar and the tensor perturbations since, in power-law inflation, the power spectrum of these perturbations are related by a constant factor [1, 3, 14].

and the general solution to this differential equation is given by (see, for e.g., Ref. [28], p. 362)

$$u_k(x) \;=\; \Biggl(A(k)\, H^{(2)}_{-\left(\beta+\frac{1}{2}\right)}\left[(\beta+1)\,\mathrm{e}^{-x}\right] + B(k)\, H^{(1)}_{-\left(\beta+\frac{1}{2}\right)}\left[(\beta+1)\,\mathrm{e}^{-x}\right]\Biggr). \quad (20)$$

The quantities $H^{(1)}_\nu$ and $H^{(2)}_\nu$ in the above solution are the Hankel functions of the first and the second kind (of order ν), respectively, and the k-dependent constants $A(k)$ and $B(k)$ are to be fixed by choosing suitable initial conditions for the modes at the beginning of inflation.

In the standard inflationary cosmology, the initial conditions are imposed on sub-Hubble scales, i.e. when the physical wavelengths $(k/a)^{-1}$ of the modes are much smaller than the Hubble radius H^{-1}. In this limit, the modes do not feel the curvature of the space-time and, hence, in terms of η, reduce to $e^{\pm ik\eta}$. The assumption that the quantum field is in the vacuum state then requires that $\mu_k(\eta)$ is a positive frequency mode at sub-Hubble scales, i.e. it has the asymptotic form (see, for instance, Ref. [27])

$$\lim_{(k/aH)\to\infty} \mu_k(\eta) \to \left(\frac{1}{\sqrt{2k}}\right)\, \mathrm{e}^{-ik\eta}. \quad (21)$$

In terms of the variable x, sub-Hubble scales (i.e. $(k/a) \gg H$) correspond to the limit $x \to -\infty$, super-Hubble scales (i.e. $(k/a) \ll H$) correspond to $x \to \infty$ and Hubble exit of the modes occur at $x = 0$. Therefore, in terms of x and u_k, the condition (21) reduces to

$$\lim_{x\to-\infty} u_k(x) \to \left(\frac{1}{\sqrt{2k}}\right)\, \mathrm{e}^{(x/2)}\, \exp -i[(\beta+1)\,\mathrm{e}^{-x}]. \quad (22)$$

This can be achieved by setting $B(k)$ to zero and choosing $A(k)$ to be

$$A(k) = \left(\frac{\pi\,(\beta+1)}{4k}\right)^{1/2} \mathrm{e}^{i\pi\beta/2} \quad (23)$$

in Eq. (20), so that we have[2]

$$u_k(x) = \left(\frac{\pi\,(\beta+1)}{4k}\right)^{1/2} \mathrm{e}^{i(\pi\beta/2)}\, H^{(2)}_{-\left(\beta+\frac{1}{2}\right)}\left[(\beta+1)\,\mathrm{e}^{-x}\right] \quad (25)$$

and it should be mentioned here that the vacuum state associated with this mode is often referred to in the literature as the Bunch-Davies vacuum [29]. The spectrum of perturbations can now be obtained by substituting the mode (25) in the expression (16) [with (η, μ_k) related to (x, u_k) by Eqs. (17) and (18)] and finally setting $x = 0$. We obtain the power spectrum, at Hubble exit, to be [30]

$$\left[k^3\, \mathcal{P}_\Psi(k)\right] = C\, \left(\frac{\mathcal{H}^2}{2\pi^2}\right) \left(\frac{k}{\mathcal{H}}\right)^{2(\beta+2)}, \quad (26)$$

where C is given by

$$C = \left(\frac{\pi}{4}\right)\, (1+\beta)^{-2(\beta+1)} \left|H^{(2)}_{-\left(\beta+\frac{1}{2}\right)}(\beta+1)\right|^2. \quad (27)$$

Note that, in obtaining the above expression, we have evaluated the power spectrum when the modes leave the Hubble radius. In the literature (see, for instance, Ref. [30]), the spectrum of perturbations is evaluated at the super-Hubble scales. These two power spectra typically differ in their amplitude by a numerical factor of order unity.

IV. CORRECTIONS TO THE STANDARD PERTURBATION SPECTRUM

As we had discussed in Section (II), in our model, the modified Wightman function of a massless field $G_{\mathrm{M}}(\tilde{x}, \tilde{x}')$ can be expressed as the difference of the Wightman functions of the massless field and a massive field of mass k_{c}, viz. $G^+_0(\tilde{x}, \tilde{x}')$ and $G^+_{k_{\mathrm{c}}}(\tilde{x}, \tilde{x}')$ [cf. Eq. (4)]. Therefore, following Eq. (15), we can define the resulting modified perturbation spectrum per logarithmic interval, viz. $\left[k^3\, \mathcal{P}_\Psi(k)\right]_{\mathrm{M}}$, as follows:

$$\begin{aligned}\int_0^\infty \left(\frac{dk}{k}\right) \left[k^3\, \mathcal{P}_\Psi(k)\right]_{\mathrm{M}} \;&=\; G^+_{\mathrm{M}}(\tilde{x}, \tilde{x}) \\ &=\; G^+_0(\tilde{x}, \tilde{x}) - G^+_{k_{\mathrm{c}}}(\tilde{x}, \tilde{x})\,. \quad (28)\end{aligned}$$

The massive field can be decomposed and quantized in terms of the normal modes along the same lines as we had done in Eqs. (12) and (14) for the case of the massless field. If we now assume that the massive field is also in the Bunch-Davies vacuum, then the modified perturbation spectrum can be written as

$$\left[k^3\, \mathcal{P}_\Psi(k)\right]_{\mathrm{M}} = \left(\frac{k^3}{2\pi^2}\right) \left[\left(\frac{|\mu_k|}{a}\right)^2 - \left(\frac{|\bar{\mu}_k|}{a}\right)^2\right], \quad (29)$$

where, as before, μ_k denotes the modes of the massless field and $\bar{\mu}_k$ denotes the modes of the scalar field with mass k_{c} which satisfies the differential equation

$$\bar{\mu}''_k + \left[k^2 + (k_{\mathrm{c}}\, a)^2 - \left(\frac{a''}{a}\right)\right] \bar{\mu}_k = 0. \quad (30)$$

Moreover, as in the standard case, the expression on the right hand side of Eq. (29) is to be evaluated at the time

[2] Using the asymptotic behavior of the Hankel function, viz. (cf. Ref. [28], p. 364)

$$\lim_{z\to\infty} H^{(2)}_\nu(z) \longrightarrow \left(\frac{2}{\pi z}\right)^{1/2} \mathrm{e}^{-i[z-(\pi\nu/2)-(\pi/4)]}, \quad (24)$$

it is straightforward to check that the function $u_k(x)$ in Eq. (25) has indeed the required limit (22).

when the modes of the massless and the massive fields leave the Hubble radius. On comparing the form of the original perturbation spectrum (16) with the modified spectrum (29), it is clear that the corrections to the standard spectrum arise as a result of the contribution due to the massive modes.

Before we proceed further with the evaluation of the corrections, we would like to stress the following point: In the standard inflationary scenario, it is well-known that the amplitude of the spectrum corresponding to the massive modes decays at the super-Hubble scales. (This can be easily shown for the case of exponential expansion, wherein the solutions to the massive modes are exactly known—in this context, see, for e.g., Ref. [31]). As we had pointed out in the previous paragraph, in our model, the trans-Planckian corrections to the standard adiabatic primordial spectrum arise due to the massive modes. Hence, within the standard inflationary picture, the amplitude of these corrections would be expected to decay at the super-Hubble scales. However, as the massive modes we have considered are supposed to represent the trans-Planckian corrections to the standard, massless modes, we shall assume that the mechanism that 'freezes' the amplitude of the standard adiabatic spectrum at super Hubble scales will also 'freeze' the amplitude of the trans-Planckian corrections at their value at Hubble exit. Hence, in what follows, we shall evaluate the corrections to the standard power spectrum when the massive modes leave the Hubble radius.

We can now introduce a new set of variables $(x, \bar{u}_k)$ which are related to the old set $(\eta, \bar{\mu}_k)$ exactly as in the massless case through the relations (17) and (18). In terms of the new variables, the differential equation (30) can be written as

$$\left(\frac{d^2\bar{u}_k}{dx^2}\right) + \omega^2(x)\,\bar{u}_k(x) = 0, \tag{31}$$

where $\omega^2(x)$ is given by

$$\omega^2(x) \;=\; \Bigg[(\beta+1)^2\, e^{-2x} - \left(\beta+\frac{1}{2}\right)^2 + \left(\frac{k_c}{\mathcal{H}}\right)^2\, \left(\frac{\mathcal{H}(\beta+1)}{k}\right)^{2(\beta+2)}\, \mathrm{e}^{-2(\beta+2)x}\Bigg]. \tag{32}$$

Unlike the massless case, the exact solution to the differential equation (31) for power-law inflation is not known. (The solution is known only for the special case of $\beta=-2$ which corresponds to exponential expansion.) Hence, we shall obtain the solution in the WKB approximation. As we shall show, for a massive field such that $k_{\rm c} \gg \mathcal{H}$, the WKB approximation turns out to be valid for all x over a range of values of β and k of our interest. For our discussion below, we shall assume that $10^{-5} \lesssim (\mathcal{H}/k_{\rm c}) \lesssim 10^{-3}$.

To begin with, note that, for $\beta \le 2$ and $k_{\rm c} \gg \mathcal{H}$, $\omega^2(x)$ remains positive for all values of x. Therefore, the WKB solutions to the differential equation (31) are given by (see, for e.g., Ref. [27])

$$\bar{u}_k^{\rm WKB}(x) = \left(\frac{1}{\sqrt{\omega(x)}}\right)\, \exp \pm i\int^x dx'\,\omega(x'), \tag{33}$$

where $\bar{u}_k^{\rm WKB}$ satisfies the differential equation

$$\left(\frac{d^2\bar{u}_k^{\rm WKB}}{dx^2}\right) + \Big[\omega^2(x) - Q(x)\Big]\bar{u}_k^{\rm WKB} = 0 \tag{34}$$

with the quantity $Q(x)$ defined as

$$Q(x) = \left[\left(\frac{3}{4\omega^2}\right)\left(\frac{d\omega}{dx}\right)^2 - \left(\frac{1}{2\omega}\right)\left(\frac{d^2\omega}{dx^2}\right)\right]. \tag{35}$$

It is then clear from Eq. (34) that the WKB solution (33) will be a good approximation to mode function $\bar{u}(x)$, only if the following condition is satisfied [27]:

$$\left|\left(\frac{Q}{\omega^2}\right)\right| \ll 1. \tag{36}$$

For ω^2 given by Eq. (32), we find that the quantity (Q/ω^2) can be written as

$$\left(\frac{Q}{\omega^2}\right) \;=\; \Bigg\{\left(\frac{5}{4\,\omega^6}\right)\left[(\beta+1)^2\,\mathrm{e}^{-2x} + (\beta+2)\left(\frac{k_c^2}{\mathcal{H}^2}\right)\left(\frac{\mathcal{H}\,(\beta+1)\,\mathrm{e}^{-x}}{k}\right)^{2(\beta+2)}\right]^2 - \left(\frac{1}{\omega^4}\right)\left[(\beta+1)^2\,\mathrm{e}^{-2x} + (\beta+2)^2\left(\frac{k_c^2}{\mathcal{H}^2}\right)\left(\frac{\mathcal{H}\,(\beta+1)\,\mathrm{e}^{-x}}{k}\right)^{2(\beta+2)}\right]\Bigg\}. \tag{37}$$

Our task now is to examine whether this (Q/ω^2) indeed satisfies the condition (36) in all the three regimes, viz.

on the sub-Hubble and the super-Hubble scales as well as at Hubble exit. Let us first consider the case of exponential expansion, i.e. when $\beta=-2$. In such a case, ω^2 and (Q/ω^2) simplify to

$$\omega^2=\left[\mathrm{e}^{-2x}+\left(k_c/\mathcal{H}\right)^2-(9/4)\right] \tag{38}$$

and

$$\left(\frac{Q}{\omega^2}\right)=\left[\left(\frac{5}{4\,\omega^6}\right)\,\mathrm{e}^{-4x}-\left(\frac{1}{\omega^4}\right)\,\mathrm{e}^{-2x}\right]. \tag{39}$$

We find that this expression reduces to

$$\lim_{x\to-\infty}\left|\left(\frac{Q}{\omega^2}\right)\right|\simeq\left(\mathrm{e}^{2x}/4\right)\to 0 \tag{40}$$

on the sub-Hubble scales, to

$$\lim_{x\to\infty}\left|\left(\frac{Q}{\omega^2}\right)\right|\simeq\left(\mathcal{H}/k_c\right)^4\,\mathrm{e}^{-2x}\to 0 \tag{41}$$

on the super-Hubble scales and to

$$\lim_{x\to 0}\left|\left(\frac{Q}{\omega^2}\right)\right|\simeq\left(\frac{\mathcal{H}}{k_c}\right)^4 \tag{42}$$

at Hubble exit. Since $(\mathcal{H}/k_c)\ll 1$, clearly, $(Q/\omega^2)\ll 1$ at Hubble exit and, therefore, the WKB approximation for the massive modes is valid for all x and k.

Let us now consider the case of power-law inflation, i.e. when $\beta<-2$. We find that, the expression (37) for (Q/ω^2) reduces to

$$\lim_{x\to-\infty}\left|\left(\frac{Q}{\omega^2}\right)\right|\simeq\left[\frac{e^{2x}}{4\,(\beta+1)^2}\right]\to 0 \tag{43}$$

on the sub-Hubble scales, to

$$\lim_{x\to\infty}\left|\left(\frac{Q}{\omega^2}\right)\right| \simeq \left(\frac{\mathcal{H}^2}{4\,k_c^2}\right)\left(\frac{k}{\mathcal{H}\,(\beta+1)}\right)^{2(\beta+2)} \times(\beta+2)^2\,e^{2(\beta+2)x}\to 0 \tag{44}$$

on the super-Hubble scales and to

$$\lim_{x\to 0}\left|\left(\frac{Q}{\omega^2}\right)\right|\simeq\left(\frac{\mathcal{H}^2}{4\,k_c^2}\right)\left(\frac{k}{\mathcal{H}\,(\beta+1)}\right)^{2(\beta+2)}(\beta+2)^2 \tag{45}$$

at Hubble exit. Evidently, the WKB approximation is valid at the sub-Hubble and the super-Hubble scales. However, at Hubble exit, the validity of the approximation depends on the values of β and k and, for a given value of β, the approximation breaks down at a sufficiently small value of k. Fig. 1 contains contour plots for $|(Q/\omega^2)|=10^{-2}$ at Hubble exit plotted as a function of β and k. It is clear from the figure that the WKB approximation will be valid for smaller and smaller values of k, provided we choose correspondingly smaller and smaller values of $-(\beta+2)$. In the limit of exponential expansion, i.e. as $\beta\to-2$, the WKB approximation, as we had shown earlier, turns out to be valid for all k.

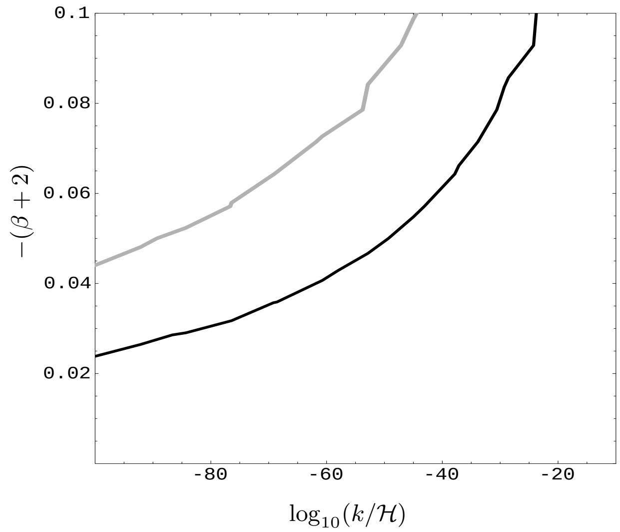

FIG. 1: Contour plots for $|(Q/\omega^2)|=10^{-2}$ at Hubble exit plotted as a function of β and k. The black contour corresponds to $(\mathcal{H}/k_c)=10^{-3}$ and the grey one to $(\mathcal{H}/k_c)=10^{-5}$. The WKB approximation is valid for values of β and k that lie below these contours. In plotting these contours we have assumed that $\mathcal{H}=10^{14}\,\mathrm{GeV}=10^{52}\,\mathrm{Mpc}^{-1}$.

A few clarifying remarks concerning the above WKB approximation for the massive modes are in order at this stage of our discussion. Even in the case of the standard massless and minimally coupled scalar field, it is well-known that, while the WKB approximation is valid at the sub-Hubble and at the super-Hubble scales, it breaks down at Hubble exit (for a detailed discussion on this point and the evaluation of the standard spectrum in the WKB approximation, see Ref. [27]). In fact, the WKB approximation will break down at Hubble exit even for a massive field if the mass of the field is smaller than (or of the order of) the Hubble scale $\mathcal{H}$. This is evident from Eq. (42) for the case of exponential expansion. However, in our model, we have assumed that the mass (viz. $k_{\rm c}$) of the field to be very large compared to $\mathcal{H}$. It is due to this reason that we find the WKB approximation to be valid at Hubble exit for all values of k in the case of exponential inflation and up to a certain minimum value of k, which depends on the values of β and $(\mathcal{H}/k_{\rm c})$, in the case of power law inflation.

The general WKB solution to the equation (31) can be written as

$$\bar{u}_k(x)\;\simeq\;\left[\left(\frac{\bar{A}(k)}{\sqrt{\omega(x)}}\right)\,\exp\,i\int^x dx'\,\omega(x') + \left(\frac{\bar{B}(k)}{\sqrt{\omega(x)}}\right)\,\exp\,-i\int^x dx'\,\omega(x')\right], \tag{46}$$

where $\bar{A}(k)$ and $\bar{B}(k)$ are k-dependent constants that are

to be fixed by the initial conditions. As mentioned earlier, we shall assume that the massive field is in the Bunch-Davies vacuum on the sub-Hubble scales. (Note that, in obtaining the standard spectrum, the massless field is assumed to be in the Bunch-Davies vacuum. Any possible excitations of the field by the evolving background are expected to be suppressed exponentially by the mass (see, for e.g., Ref. [32]). Therefore, it is natural to assume that the massive field—in particular, a field with a mass (viz. $k_{\rm c}$) much greater than the Hubble scale—is in the Bunch-Davies vacuum as well.) Hence, the modes $\bar{u}_k$ are required to have the limiting form (22) which leads to the conditions that $\bar{B}(k)=0$ and $\bar{A}(k)=[(\beta+1)/2k]^{1/2}$. Therefore, the mode $\bar{u}_k(x)$ is given by

$$\bar{u}_k(x) \simeq \left(\frac{\beta+1}{2k\,\omega(x)}\right)^{1/2} \exp\, i\int^{x} dx'\,\omega(x') \tag{47}$$

and, using these modes, it is straightforward to evaluate the modified perturbation spectrum (29). We find that the resulting spectrum can be written as

$$\begin{aligned}\left[k^3\,\mathcal{P}_{\Psi}(k)\right]_{\rm M} \;\simeq\; C\,\left(\frac{\mathcal{H}^2}{2\pi^2}\right)\,\left(\frac{k}{\mathcal{H}}\right)^{2(\beta+2)}&\\ \times\left[1-\bar{C}\,\left(\frac{\mathcal{H}}{k_c}\right)\,\left(\frac{k}{\mathcal{H}}\right)^{(\beta+2)}\right]&, \end{aligned}\tag{48}$$

where $\bar{C}$ is given by

$$\bar{C}=\left[2\,C\,\left(\beta+1\right)^{3(\beta+1)}\right]^{-1}. \tag{49}$$

The modified spectrum (48) has some similarities to the power spectrum that has been obtained recently from non-commutative inflation [18] and the spectrum exhibits a suppression of power at the large scales. In order to illustrate this feature, we have plotted the modified spectrum (48) as well as the standard spectrum, normalized to $\mathcal{H}^2$, in Fig. 2 below. (Note that, in plotting the modified spectrum, we have chosen the values of $(\mathcal{H}/k_{\rm c})$ and β so that the WKB approximation is valid for the massive modes over the range of k of interest.) It is evident from the figure that the modified spectrum exhibits a suppression of power around $\log_{10}(k/\mathcal{H})\sim -56$ which corresponds to $k\sim 10^{-4}\,{\rm Mpc}^{-1}$ or $l=2,4$—a feature that seems to be necessary to explain lower power in the quadrupole ($l=2$) and the octopole moments ($l=4$) in the CMB (see, for e.g., Ref. [33]). Though the modified spectrum we have obtained (by using the standard inflationary parameters) shows a suppression of power around the expected values of k, the extent of the suppression proves to be far less than that is required by the WMAP data. In order to fit the WMAP data, one seems to require a spectrum of the following form (see Ref. [17]; in this context, also see Ref. [34]):

$$\left[k^3\,\mathcal{P}_{\Psi}(k)\right]_{\rm M}=A_{\rm s}\,k^{(n_{\rm s}-1)}\left[1-\exp-\left(k/k_{*}\right)^{\gamma}\right], \tag{50}$$

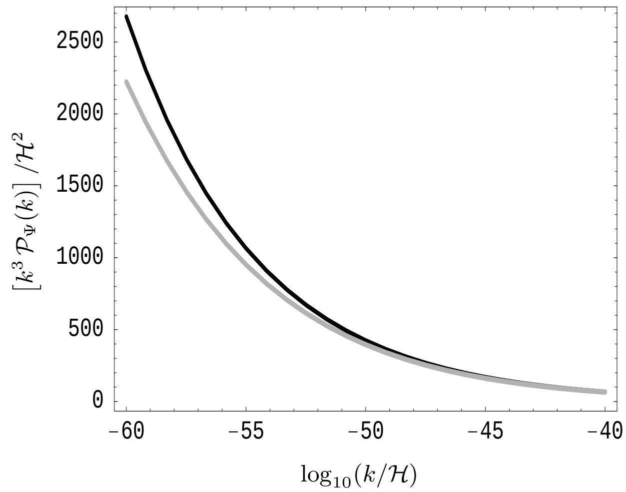

FIG. 2: Plots of the standard power spectrum $\left[k^3\,\mathcal{P}_{\Psi}(k)\right]$ (black curve) and the modified power spectrum $\left[k^3\,\mathcal{P}_{\Psi}(k)\right]_{\rm M}$ (grey curve), normalized to $\mathcal{H}^2$. The grey curve corresponds to $(\mathcal{H}/k_{\rm c})=10^{-3}$ and $\beta=-2.04$—values that have been chosen so that the WKB approximation is valid around $k\sim 10^{-4}\,{\rm Mpc}^{-1}$. In plotting these spectra we have assumed that $\mathcal{H}=10^{14}\,{\rm GeV}=10^{52}\,{\rm Mpc}^{-1}$.

where $A_{\rm s}$ and $n_{\rm s}$ are the amplitude and index of the standard spectrum, $k_{*}\simeq 5\times 10^{-4}\,{\rm Mpc}^{-1}$ and $\gamma\simeq 3.35$. To illustrate the fact that our model predicts far less suppression than is required by the observations, it is useful to consider the following ratio of the modified spectrum and the standard spectrum:

$$\mathcal{R}(k)=\left(\frac{\left[k^3\,\mathcal{P}_{\Psi}(k)\right]_{\rm M}}{\left[k^3\,\mathcal{P}_{\Psi}(k)\right]}\right). \tag{51}$$

In Fig. 3 below, we have plotted the ratio $\mathcal{R}(k)$ for the cases of the modified spectrum predicted by our model [viz. (48)] and the modified spectrum (50) that seems to be required to fit the observations. [Note that the ratio $\mathcal{R}(k)$ for these spectra is given by the expressions within the square brackets in Eqs. (48) and (50).] It is evident from the figure that, in power law inflation, the suppression of power predicted by our model for the scalar power spectrum is far less than that is required to fit the WMAP data.

We would like to stress here the following points regarding the modified spectrum we have obtained. Firstly, one would tend to assume that the high energy effects will leave their imprints only at the ultra-violet end of the primordial perturbation spectrum. In contrast, we find that, what are supposedly trans-Planckian effects, result in a modification of the spectrum at the infra-red end. This essentially arises due to the fact that the longer wavelength modes leave the Hubble radius at earlier epochs

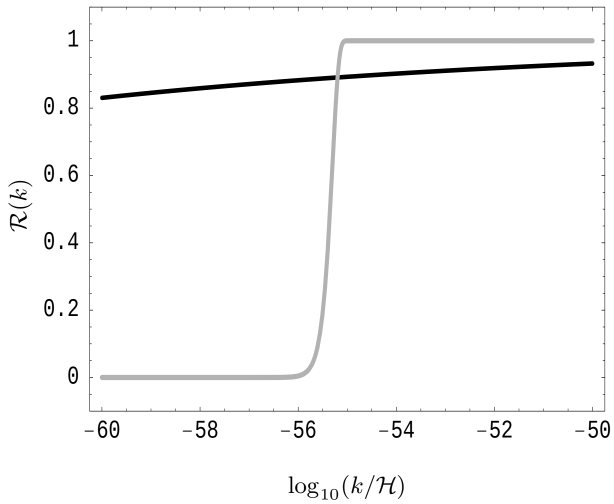

FIG. 3: The ratio $\mathcal{R}(k)$ of the modified to the standard spectrum plotted for the modified spectrum predicted by our model [viz. (48)] and the spectrum (50) that seems to be required to fit the WMAP data. The black curve corresponds to the modified spectrum (48) for $(\mathcal{H}/k_c) = 10^{-3}$ and $\beta = -2.04$—values that have been chosen so that the WKB approximation holds good around $k \sim 10^{-4}\,\mathrm{Mpc}^{-1}$. The grey curves corresponds to the spectrum (50) for $k_* = 5 \times 10^{-4}\,\mathrm{Mpc}^{-1}$ and $\gamma = 3.35$. Note that, in plotting these curves we have assumed that $\mathcal{H} = 10^{14}\,\mathrm{GeV} = 10^{52}\,\mathrm{Mpc}^{-1}$.

thereby carrying the imprints of the high energy effects[3]. Secondly, assuming that the Planck scale effects can be expected to improve the divergence structure of quantum field theory, we had expressed the modified Wightman function (4) as the *difference* of the massless and the massive Wightman functions. Due to this reason and, also since the correction term to the power spectrum due to the massive modes is positive definite [cf. Eq. (29)], it is clear that the corrections will always suppress the power rather than enhance it. However, evidently, the actual form and the extent of the suppression will depend on the correction term. Though, interestingly enough, we find that the massive modes indeed suppress the power at large scales as required by the observations, the amount of suppression proves to be *far less* than that is needed to fit the WMAP data. Fourthly, we should point out that our model does not contain any free parameters other than the ratio $(\mathcal{H}/k_{\rm c})$ and, hence, it can be said to predict the loss of power at large scales naturally. Finally, as we had mentioned earlier, we have evaluated the trans-Planckian corrections to the standard, adiabatic spectrum when the massive modes leave the Hubble radius. In the standard inflationary picture, the amplitude of the spectrum of the massive modes would decay—the correction term in the spectrum (48) would retain its spectral shape, but its amplitude would decay as $e^{3(\beta+1)x}$—at the super Hubble scales (i.e. as $x \to \infty$). Since, the massive modes in our model represent trans-Planckian corrections to the standard massless modes, we have assumed that the mechanism that 'freezes' the amplitude of the standard spectrum at super Hubble scales will also 'freeze' the amplitude of the corrections at their value at Hubble exit.

[3] We thank Robert Brandenberger for drawing our attention to this point.

V. DISCUSSION

In this work, we have studied the trans-Planckian effects on the spectrum of the primordial density perturbations in the power-law inflationary scenario using an approach that preserves local Lorentz invariance. Motivated by the fact that quantum gravitational effects can be expected to improve the divergence structure of standard quantum field theory, we *assumed* that the trans-Planckian physics modifies the standard massless propagator such that the modified propagator can be expressed as the *difference* of the original massless propagator and a massive propagator with a mass of the order of the Planck mass.

In the standard inflationary scenario, the primordial perturbation spectrum is determined by the Fourier transform of the massless scalar field propagator. Therefore, modifying the scalar field propagator leads to modifications in the perturbation spectrum. We find that, in our model, the resulting modified spectrum remains scale invariant at the ultra-violet end, but, interestingly, it exhibits a suppression of power at the infra-red end—a feature that seems to be necessary to explain the low quadrupole and octopole moments measured in the CMB [15, 17, 33, 34]. However, at Hubble exit, the amount of suppression predicted by our model in power-law inflation turns out to be *far less* than as expected from WMAP data. Nevertheless, the loss of power at small k suggests that the power spectrum we have obtained may fit the WMAP data better than the standard ΛCDM model. It will be interesting to analyze the implications of the WMAP data for our model in the context of slow-roll inflation [35].

Naively, one would expect that very high energy effects will leave their imprints only at the ultra-violet end of the primordial spectrum. However, we find that the high energy effects lead to a modification of the spectrum at the infra-red end. As we had pointed out in the last section, this can be attributed to the fact that the longer wavelength modes leave the Hubble radius at earlier epochs thereby carrying the signatures of the high energy effects.

Finally, we would like to emphasize a very attractive feature of our approach which can result in corrections at the infra-red *and* the ultra-violet ends of the perturbation spectrum. Recall that in obtaining the modified spectrum (48) we had assumed that both the massless and the massive fields are in the Bunch-Davies vacuum.

This need not be the case. While it is natural to assume that the massive field is in the Bunch-Davies vacuum (in particular, a very heavy field with a mass of the order of the Planck mass), one can assume the massless field to be in a state such as the minimal uncertainty state [10]. In such a case, the resulting modified spectrum will have corrections at both the ends. While at the infra-red end, we will simply reproduce the corrections we have obtained, in the ultra-violet limit, the corrections will be of the form that has been recently obtained in the literature [11].

Acknowledgments

The authors wish to thank T. Padmanabhan and Robert Brandenberger for their comments on an earlier version of the manuscript. LS wishes to thank Namit Mahajan for discussions. LS would also like to thank the High Energy Physics Group of The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy for hospitality, where this work was completed.

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